Journal of Automation and Information Sciences, 43(4), 2011

Review on the book by M.V. Sinkov, Yu.E. Boyarinova, Ya.A. Kalinowski "Finite-dimensional Hypercomplex Numerical Systems. Foundations of the Theory. Applications", Infodruk, Kiev, 2010

V.M. Kuntsevich

[1]*Academician of National Academy of Sciences of Ukraine, doctor of technical sciences, professor, principal researcher of Institute of Space Research of National Academy of Sciences of Ukraine and National Space Agency of Ukraine, Kiev.*

ISSN 1064-2315

Despite the fact that some problems of the theory of finite-dimensional hypercomplex number systems have already been known and have numerous applications in the last fifty years, though there is reason to believe the book by M.V. Sinkov, Yu.E. Boyarinova, Ya.A. Kalinowski “Finite-dimensional hypercomplex numerical systems. Foundations of the theory. Applications”, is a significant development of the theory of representation and information processing. The book is well arranged, such important problems as the systems multiplicity, isomorphism of systems, construction of analytic functions and some other problems are specified.

One of the major problems while conducting remote sensing of the Earth, especially while obtaining images of the surface of the Earth with high resolution is a problem of precision motion control of a spacecraft, considered as a solid, in the orbit,. This problem has been solved, and is solved by many authors using various forms of construction of algorithms, namely, vector, coordinate, matrix. In fact, all these forms are equivalent.

At present, the algorithms with the matrices of direction cosines are still the most common. Such class of algorithms, as applied to the problem of control of a spacecraft, has a number of significant shortcomings, namely: the trigonometric functions are computed slowly and with respect to the areas, which are close to the Earth poles, become degenerate. This was the reason why the creators of the first control systems of spacecrafts V.N. Branets and I.P. Shmyglevsky (joint venture firm of S.P. Korolev) undertook an active development of algorithms based on the quaternion equations, more convenient for these control problems. As the basis the use was made of the geometric constructions, associated with quaternions, introduced first by the author of the quaternion algebra W. Hamilton. Quaternions enable in a convenient way to set turns in the three-dimensional space, which is especially important for description of rotational motion of a rigid body. By means of the quaternion instrumentality the problem of determining the parameters of a rigid body turn and problem of adding turns are solved very efficiently.

The application domain of quaternions can be even greater, given the authors’ suggestion of replacing the real coefficients at imaginary units of the quaternion by complex, double, and dual. This gives three hypercomplex numbers — biquaternion, elliptic biquaternion and parabolic biquaternion. This will enable the problems with the screw motion of robots and manipulators of complex structures to be solved more effectively.

This review does not attempt to analyze all the studies presented in the book. This is a great and laborious work. Therefore, let us dwell only on some questions, among which enumeration and classification are important.

The authors correctly raise and solve the question on enumeration of hypercomplex number systems. Moreover, if the quaternion is a noncommutative extension of complex numbers field, then there is also a commutative extension — it is quadriplex numbers. Enumeration of hypercomplex number systems, with identifying the properties of the commutativity, associativity, the presence of unity in the basis, and many other things, is a difficult but very necessary task of classification of hypercomplex number systems (HNS).

Enumeration and classification of HNS are the first steps in the study of the structure of the HNS set. By this it is ment whether HNS is a direct sum of fields of real, complex numbers and quaternions? Is there a unit in a basis, of what kind are the zero divisors, conjugate numbers, norms, and many other questions?

Answers to these questions are given in the reviewed book, where basic mathematical operations are consistently presented, and considerable attention is paid to the multiplication operation. The multiplication matrix is an original “indicator” of the hypercomplex number system and characterizes it,

allowing one to identify individual properties and characteristics of the investigated HNS. The most complete information about the multiplication table allows one to draw conclusions about the potential computing performance while desire for its increase.

The book presents materials on the definition of the analyticity of the functions of hypercomplex argument and the analyticity of the specific functions is considered, as well as some questions of construction of differential equations from a hypercomplex argument and methods of their solving.

The authors paid attention to the construction of the toolbox containing a set of algorithms and software for operations in hypercomplex number systems, that greatly simplifies the design of the considered systems.

Another important area of HNS application is a problem of secret division while transferring an encrypted text, formulated and solved by the authors with the use of hypercomplex presentation of data. In this case, the increase of the stability parameter is reached, which is important while solving cryptographic problems.

The above indicates the unconditional urgency of conducted research for the theory and practice and is a fundamental development of the theory of representation and processing of information.